Visualization and Extraction of Carvings for Heritage Conservation

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Abstract—We present novel techniques for visualizing, illustrating, analyzing, and generating carvings in surfaces. In particular, we consider the carvings in the plaster of the cloister of the Magdeburg cathedral, which dates to the 13th century. Due to aging and weathering, the carvings have flattened. Historians and restorers are highly interested in using digitalization techniques to analyze carvings in historic artifacts and monuments and to get impressions and illustrations of their original shape and appearance. Moreover, museums and churches are interested in such illustrations for presenting them to visitors. The techniques that we propose allow for detecting, selecting, and visualizing carving structures. In addition, we introduce an example-based method for generating carvings. The resulting tool, which integrates all techniques, was evaluated by three experienced restorers to assess the usefulness and applicability. Furthermore, we compared our approach with exaggerated shading and other state-of-the-art methods.

Index Terms—Feature extraction, heritage preservation, Frangi filter, surface analysis, feature filtering

1 INTRODUCTION

In the 13th century, artists have carved illustrations of the episodes from the church history and depictions of important persons into the plaster of cathedrals and churches. One prominent example are the carvings on the walls of the cloister of the Cathedral of Saints Catherine and Maurice in Madgeburg. Due to the open structure of the cloister, these carvings were subjected to weathering. Nowadays, the illustrations in parts of the plaster are hard to recognize and many structures are only visible with extreme side light. Therefore, restorers and historians are in need of a software tool that allows to analyze digital copies of the plasters, and to explore and edit the carvings virtually. In addition, virtual images visualizing the carvings and illustrations of the original shape of the carvings are attractive for visitors of the historic site.

In this paper, we propose techniques addressing the technical challenges that have to be solved for creating such a tool and we present an evaluation of the associated software by the relevant target group.

We want to contrast our approach from methods for general sunken-relief extraction. The carvings we consider are special since the structures are canal-like and flat. Aging and weathering have additionally flattened the structures so that they can barely be perceived with the naked eye. Furthermore, the surface of the plaster exhibits irregular structures and the data contains noise. The amplitude of the noise and the height of the structures almost match the depth of the carvings. Both factors complicate the extraction of the carving structures.

To enable the processing of sunken-relief extraction, we first digitalize the plasters and carvings using a passive stereo vision system, resulting in a triangle mesh approximating the surface of the plaster. For analyzing the digitized plaster, we developed a detection filter for carvings in surfaces that searches for parts with a distribution of principal curvatures that exhibits characteristic for carvings, i.e., locally cylinder-like structures. This approach is based on the Frangi filter [9] from medical image processing, which is used for detecting vessel-like structures in images. We adapt this method to detect carvings in 3D surfaces. Our method is specifically designed for detecting canal-like structures and is robust against noise. For filtering the set of detected carvings structures and selecting the most salient structures, we propose a scheme that combines connected component analysis and an importance measure for the saliency of the carvings. Finally, visualization techniques are employed to emphasize the detected carving structures, see Figure 1. The software implementing these methods has been tested by domain experts in a user study that highlights the effectiveness and utility of our approach. Our experiments demonstrate that state-of-the-art methods for relief extraction, as well as feature line extraction and exaggerated shading, fail on the type of data we are considering. In summary, we make the following contributions:

• An algorithm for detecting carving structures in surfaces based on an adaptation of the Frangi filter and a scheme for ordering connected components of the detected carving structures by saliency

• A framework for interactive exploration of carving structures in surfaces that allows for selecting and deselecting structures, adding new carvings and visually highlighting them

2 RELATED WORK

For the illustration of carvings, we analyze the field of illustrative visualization. One relevant topic in this field is the depiction of strong features via lines.

Feature Lines Methods for feature line extraction can be divided into view-dependent and view-independent approaches. Although, for carving extraction it is important to illustrate all features independent of the camera position, it is worth mentioning view-dependent approaches as well. Ridge and valley lines have the property that every point is an extremum of the principal curvature on the corresponding curvature line [15,29]. The computation of these lines is delicate since it involves estimating third-order derivatives on the surfaces. Prominent approaches use surface fitting [28,39] or a combination of discrete curvature approximation and smoothing techniques [14]. A technique for view-dependent feature line detection was introduced by DeCarlo et al. [7]. They extended the contour definition to determine salient features. Another view-dependent feature line approach was presented by Xie et al. [38]. Their method depends on the underlying surface shading and allows for compensating noise by adding light sources. This approach was later improved [41]. A method that combines ridges and valleys with a view-dependent approach was proposed by Judd et al. [16]. They defined view-dependent curvature quantities to apply their technique. Kolomenkin et al. [18] proposed a view-independent approach to determine the transitions from flat regions to ridges. Their approach can be used to enhance small features on archeological objects. Zhang et al. [42] introduced Laplacian lines, which are used to illustrate prominent features. The Laplacian of the surface normal field is pre-computed and the result is used for illustrating the features. Weinkauf and Günther [37] used the Morse–Smale complex of a curvature-based indicator function to construct networks of feature lines. Robustness against noise is improved by simplifying the Morse-
with the original ones: the larger the differences, the more salient based on curvature and material characteristics. An exaggeration technique called Radiance Scaling by enhancing features on the surface. Vergne et al. [36] presented a technique to automatically process scanned artifacts. They defined a prominent surface shape. For analyzing salient features on the surface make it difficult for their method to extract the carvings.

Here, the surface is first segmented using a curvature-based watershed algorithm. This leads to a reliable estimation of the intrinsic and extrinsic camera parameters. Testing the spatial intersection of the calibrated stereo vision system using a stochastic pattern on a plane surface (calibration standard) results in residual errors up to a maximum of 0.02 mm, which is acceptable for this application.

To compute pairs of corresponding points in both images of one capture, we start with an evenly distributed field of points on one image. Then the task is to find the best correlation between a squared mask around that point on image one and the projection of this mask along the epipolar line on image two. To save computation time and to increase robustness, this line is trimmed by taking into account the depth of field (sharpness). Because of the slightly different viewing angles and varying imaging characteristics, the correlation algorithm has to be robust against differences in dynamics and the average intensity. Along with other approaches, the normalized cross-correlation introduced by Moravec [27] gives reliable results. It is invariant against additive and multiplicative intensity changes in the search window. The spatial intersection for all rays of corresponding points for one capture leads to a resulting 3D point cloud of up to 4 million points.

To reconstruct a larger area, we have to combine the results of a number of captures. This is accomplished by manually picking three corresponding 3D points within two patches and a subsequent refinement using the Iterative Closest Point algorithm [3]. Since the

Feature-based shading A technique for enhancing surface features by modifying the surface normals was presented by Cignoni et al. [5]. Their idea is to smooth the normals and to compare them with the original ones: the larger the differences, the more salient the underlying geometric feature. Another approach for enhancing geometric features by modifying the surface normals was introduced by Rusinkiewicz et al. [32]. They iteratively smooth the surface normals and store the results. Afterwards, the stored sequences of normals are combined to determine the shading. Lee et al. [23] used an approach that automatically determines lighting positions to enhance features. Here, the surface is first segmented using a curvature-based watershed method. These patches are used for the placements of lights. Vergne et al. [35] introduced a local shape descriptor that enhances salient features. They warped the incoming lighting in order to increase the view-dependent surface curvature information. This leads to a perceptually enhanced surface shape. For analyzing salient features on archaeological surfaces, Kolomenkin et al. [20] presented an approach to automatically process scanned artifacts. They defined a prominent field, which is a smooth field with certain properties at surface features to enhance geometrical features. Vergne et al. [36] presented a technique called Radiance Scaling that improves the classical shading by enhancing features on the surface. They scaled light intensities based on curvature and material characteristics. An exaggeration technique to enhance relief structures was proposed by Miao et al. [25]. This approach is based on three features maps comprising local height, normal difference and mean curvature. Furthermore, the normals are adjusted to gain an enhanced rendering of the surface. Ammann et al. [2] introduced an approach that analyzes several surface features per vertex. These features are used to combine different shading techniques to enhance salient structures on the surface. As this approach enhances all features, it may be prone to noise and even highlight these undesired regions. For the illustration of complex archaeological artifacts, Gilboa et al. [12] reported on the development of computerized automatic systems. Through automatic feature detection, they highlighted salient areas as line drawings, colored images or enhanced 3D models. Sharma and Kumar [33] presented a way to investigate artifacts with machine-learning techniques. Their system employs the user’s preferred visualization settings to generate a reasonable result. Several techniques that depend on various vertex features, e.g., curvature, ambient occlusion factor, are applied to gain an appropriate shading result.

Relief extraction and generation A large body of literature on relief extraction and generation and we would like to point to the survey by Kerber et al. [17] for an overview. A general introduction to 3D shape analysis for archaeological surfaces was provided by Tal [34]. Closest related to our work is the relief extraction scheme by Zatzarinni et al. [40]. They represented the surface containing the relief as a height field over a base surface. The relief is found through an analysis of the height field. In contrast to our scheme, their method aims at reliefs with enough heights (or depth). In the case of the carvings considered here, the ratio between depth of the reliefs, amplitude of noise and natural structures in the surface and the (very low, but present) curvature of the surface make it difficult for their method to extract the carvings.

3 DATA ACQUISITION
The incisions cover an area of 37 m by 5 m outside on a wall to the courtyard of the Magdeburg cathedral. Because of current restoration work, we could use an existing scaffold for data capturing. Varying lighting conditions and unavoidable vibrations of the scaffold require a rapid measurement process. We decided to use a passive stereo vision system because it requires only a pair of synchronously captured images.

The measurement area was chosen based on the required resolution of less than 0.1 mm. Using two grayscale cameras with a resolution of 2048 × 2048 pixels the system was designed to capture an area of about 170 mm by 170 mm, see Figure 2. The calculation of calibration parameters was accomplished with the OpenCV library. We used a precisely measured calibration plane with circular marks. The centers of the images of the circular marks can be localized with high accuracy. This leads to a reliable estimation of the intrinsic and extrinsic camera parameters. Testing the spatial intersection of the calibrated stereo vision system using a stochastic pattern on a plane surface (calibration standard) results in residual errors up to a maximum of 0.02 mm, which is acceptable for this application.

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Fig. 1. On the left the original model with standard headlight. On the right the same dataset with our visualization technique.
we decided to digitalize the whole plaster with high resolution and to wall with 2.5 m in length.

For the analysis and processing of carvings in plasters, we developed methods that can emphasize the carvings. The experts are interested in interactively modifying and controlling the visualization, e.g., by selecting additional structures to be emphasized or manually removing falsely detected carving structures. Since with standard visualization of the scan data, it is still difficult to see carvings, the experts asked for visualization techniques that allow the user to modify the set of carving structures, e.g., erasing wrongly detected carvings or adding carvings.

In the real-world plaster, the carving can barely be perceived with the naked eye. Only under special lighting conditions, the restorers can get an impression of the carving structures. To circumvent these difficulties, we decided to digitalize the whole plaster with high resolution and to allow the domain experts to process the data such that the carvings can be illustrated. Since with standard visualization of the scan data, it is still difficult to see carvings, the experts asked for visualization methods that can emphasize the carvings. The experts are interested in interactively modifying and controlling the visualization, e.g., by selecting additional structures to be emphasized or manually removing falsely detected carving structures.

### 5.1 Carve Extraction and Ranking

Though the plaster that we want to analyze is attached to one wall of the cloister, it is not planar. Therefore, the carvings cannot be described by a height field over the plane. We treat the digitalized plaster as a curved surface, in which we want to detect the carvings.

Curvature computation For detecting carvings, we analyze the distribution of the curvatures over the surface. Since we are working with triangle meshes, we need to compute the principal curvatures and curvature directions of triangle meshes. Following Rusinkiewicz [31], we first compute a discrete shape operator in every triangle, which measures the deviation of the normals across the edges of the triangle. Then, a discrete shape operator at the vertices is constructed by transforming and averaging the shape operators of the triangles. The eigenvalues of the shape operator are the principle curvatures $\lambda_1, \lambda_2$, which are ordered such that $|\lambda_1| \leq |\lambda_2|$. When computing the discrete shape operator (in particular for noisy data), it is recommended to smooth the surface beforehand. We used bi-Laplacian smoothing, which results in a robust curvature computation in our experiments. An alternative to smoothing the surface is to use larger neighborhoods for the curvature computation, see [6, 13].

Frangi filter for surfaces The carving structures we want to extract are “small canals” in the shape. At the bottom of each canal, the geometry equals that of a round tube or a cylinder. This means one principal curvature vanishes and the other one is approximately one over the radius of the cross section of the carving, i.e., $\lambda_1 = 0$ and $\lambda_2 \neq 0$. Figure 3 shows examples of surfaces and their principal curvatures. Since the carvings are engraved, the non-vanishing principal curvatures of all carvings have the same sign. Whether the sign is positive or negative depends on the orientation of the surface. We choose the orientation such that the surface normals point outwards (out of the wall) and the curvature values are positive. For detecting the carving (or vessel) structures, we use, analogous to the Frangi filter [9], the quantities $\mathcal{F} := \sqrt{\lambda_1^2 + \lambda_2^2}$ and $\mathcal{R}_g := \lambda_1 / \lambda_2$ and define the vesselness

patches overlap by about 40%, there are enough unique features in the point set to ensure a robust registration. A drawback of this method is that the overall shape of the wall becomes slightly deformed after registration of a large number of patches. This is not affecting the visual perception of the result, but a comparison to point data captured by 3D laser scanning shows a significant deviation of several centimeters on a wall with 2.5 m in length.

4 Requirement Analysis

Based on a literature search and discussions with domain experts, we devised the following list of requirements:

- A method for detecting carving structures
- Interaction techniques that allow the user to modify the set of carving structures, e.g., erasing wrongly detected carvings or adding carvings
- A visualization technique that enhances the carvings

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5 Extraction and Generation of Carvings

For the analysis and processing of carvings in plasters, we developed schemes for carving extraction and ranking as well as methods for generating new carvings.

Fig. 2. Stereo camera setup (left) and corresponding images (middle) for the computation of a 3D point cloud (patch). On the right the registration result for nine false-colored patches is shown.

Fig. 3. Different greyscale images are illustrated with their corresponding height surfaces. The height surfaces’ principal curvatures are listed below.

(a) $\lambda_1 = \lambda_2 > 0$
(b) $\lambda_1 = 0, \lambda_2 > 0$
(c) $\lambda_1 < 0, \lambda_2 > 0$
(d) $\lambda_1 = 0, \lambda_2 < 0$
(e) $\lambda_1 = \lambda_2 < 0$

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we rank the components by their saliency. For extracting the carvings, we use a blue to cyan colormap to display the influence of the parameters $\beta$ and $c$. To depict the threshold, we additionally displayed all values that are smaller than the threshold in black.

Parameter adjustment The vesselness measure, Eq. 1, involves two parameters, $\beta$ and $c$. To help the users in choosing values for the two parameters, we create preview galleries. The users can choose preferred parameter settings by selecting an image. An example of an image gallery is shown in Figure 4. After a preview image has been selected, the corresponding parameter setting can be modified to fine-tune the result. Our user-interface for this step is inspired by the work of Münch et al. [26]. When pressing the right mouse button, the user can alter the two parameters by mouse movements. Horizontal movements affect only one of the parameters and vertical movement allows the user to fine-tune the result. The colormap we use for the non-selected components ranges from dark red to gray over orange, see Figure 5. Parts of the surface that do not belong to a carving structure receive a color reflecting its saliency (cf. Eq. 2). The colormap we use for the non-selected components ranges from dark red to gray over orange, see Figure 5. The colormap is shown at the bottom, where values range from gray (minimum value) to red (maximum value). The minimum, maximum value correspond to the minimum, maximum saliency (cf. Eq. 2) in the set of detected components. The values in between are linearly interpolated. The user can activate the $n$ components that will be enhanced in the rendering. The selection proceeds in two phases, first the user specifies a number $n$ of components to be selected. Our tool then selects the $n$ component with highest saliency value. The result for $n = 4$ is shown in (c). Activated components are shown in cyan. To fine-tune the result, the user can further modify the selection set. In the final rendering, the selected carvings are emphasized (d).

Activating the carvings After connected components in the carvings structures are determined and the components are ranked by saliency, the user can use this information for selecting the components that will be enhanced in the rendering. The selection proceeds in two phases, first the user specifies a number $n$ of components to be selected. Our tool then selects the $n$ component with highest saliency value. In the second phase, the user has full control over the selection and can manually add components to and remove components from the selection. To help the user in modifying the selection, our tool assigns colors to all components. Selected components are displayed in cyan and non-selected components receive a color that reflects their saliency value (cf. Eq. 2). The colormap we use for the non-selected components ranges from dark red to gray over orange, see Figure 5. Parts of the surface that do not belong to a carving structure receive a dark color. To further assist the user during the manual modification of the selection set, already activated components will be highlighted in red by a mouse over to indicate that clicking will delete the selection and not activated components will be colored in green to indicate that clicking will activate them.

Visualizing the carvings To emphasize the selected carving structures in the rendering, the shading of the selected components is modi-
which the carvings are represented using a height field. For this, we
with a similar profile. The first step is to construct a base surface over
the original surface. The height
r
Laplacian smoothing, i.e., mean curvature flow. The implicit smoothing
the scheme described in Sec. 5.1. The generative model represents the
carvings, we analyze the existing carvings, which are extracted using
in style to the ones that are already in the data.

Fig. 6. First, the carving is smoothed and the boundary is fixed (top row). Afterwards, every point on the smoothed carving is assigned the height
to the original carving. The values are ordered according to
their height and plotted. Finally an exponential function is fitted to the
ordered height data.

fied. The idea is to use the vesselness scalar field to weight the lighting
in order to enhance the carving cues. Let \( I \) denote the light vector. Then
the shading is computed using \( s = (I, n)^{1+\alpha} \), where \( \alpha \) expresses
how strongly the carving cues are emphasized. We used \( \alpha = 2 \) for
all results. Using a significantly higher value of \( \alpha \) would result in
stronger, but unnatural looking carvings. In addition to emphasizing
the selected carvings, our tool allows users to de-emphasize the non-
selected structures. This is achieved by isotropically smoothing the
normals of the surface (using 10 iterations of Laplace smoothing) and
using the smoothed normal vectors to compute the shading everywhere
except for the selected carving structures.

5.2 Carve Generation

In addition to visualizing the carvings, we want to provide a tool for
generating carving structures. For example, to help with the recreation
of lost parts of an illustration by adding new virtual carvings. Since
editing the surface itself is a challenging problem which requires sophis-
ticated modeling tools [10], we decided to preserve the original surface
and add virtual carvings overlaying the original plaster. This is realized
by modifying the surface normals in the areas where new carvings are
added. To specify the location and width of the virtual carvings the user
is provided with a brush tool. The size of the brush specifies the width
of the carving. The result of the brushing is stored in a scalar field on
the surface. Initially, this scalar field is 0 for every vertex. Whenever
the brush is used, every vertex inside the brushed region in screen space
is assigned a new scalar value. Denoting the distance of a vertex to
the brush is used, every vertex inside the brushed region in screen space
is assigned a new scalar value. Denoting the distance of a vertex to
the brush by
\( \text{dist} \) we assign the value \( r_n = \text{dist} \), which
ensures that the midpoint assumes the maximum value. Whenever
the user draws over existing brushed vertices, the already stored values
are compared with the potential new values. The greater value is then
assigned to the vertex. We use a scalar function \( f(r_n) \) to determine the
modified normals of the surface. The function serves as a desired height
(or offset in normal direction) over the original surface. The height
value at any point depends only on the value \( r_n \) assigned to the point
during brushing. The function \( f \) depends on the carvings observed in
the data. The goal is to make the newly created carvings look similar
in style to the ones that are already in the data.

Height Field Calculation To obtain the generative model for new
carvings, we analyze the existing carvings, which are extracted using
the scheme described in Sec. 5.1. The generative model represents the
carvings as a height field over a smoothed version of the surface. The
profile of the height field is analyzed and used to generate new carvings
with a similar profile. The first step is to construct a base surface over
which the carvings are represented using a height field. For this, we
copy the surface and heavily smooth the carvings by applying implicit
Laplacian smoothing, i.e., mean curvature flow. The implicit smoothing
is defined by the iteration

\[
p_i = (1 - \mu L)p_{i+1}, \quad p \in \mathcal{Y}_\epsilon,
\]

where \( I \) is the identity matrix, \( L \) is the Laplacian matrix, and \( \mu \) is
the weighting factor, which we set to 0.05. The iteration starts with
the original vertex positions \( p_0 \). After each smoothing step, the Eu-
clidean distance between the new and the old positions of the vertices is
computed

\[
D_{i+1} = \sum_{p \in \mathcal{Y}_\epsilon} \|p_{i+1} - p_i\|
\]

The smoothing flow converges when the quotient \( D_{i+1}/D_i \) tends to 1.
We stop the smoothing if \( D_{i+1}/D_i > 0.99 \). In practice, the smoothing
process is terminated after 30-50 iterations. During the smoothing, the
carvings evolve into their surface normal directions and towards the
minimal surface spanned by the constrained boundaries.

We measure the distance of the carving to the minimal surface, see
Figure 6. Explicitly, we look for the intersection of the line generated
by the point on the smoothed carving and the new normal with the
triangles on the original carving. The intersection point and the point
on the smoothed carving yield the height distance. In case two or more
intersections occur, we assign the shortest distance to the point. Every
point of the smoothed carving is assigned to a distance, i.e., the height
scalar field. For every carving component, we scale the distance to the
\( x \)-axis in a range of \([0,0.5]\).

Then a Gaussian distribution \( f(x) = a \cdot \exp\left(-((x-0.5)/c)^2\right) + d \)
in the range \( x \in [0,0.5] \) is fit to the data, see Figure 6 bottom right. The
resulting function can be used as the input for the drawings on the
surface: \( f(r_n) \). This yields the scalar field \( \mathcal{E} \). Based on this scalar field \( \mathcal{E} \),
we determine the gradient per triangle. Given a triangle \( \Delta = \{i,j,k\} \),
the gradient of every triangle is determined by:

\[
\nabla g = (\mathcal{E}_j - \mathcal{E}_i) \frac{(p_j - p_i)}{2A_\Delta} + (\mathcal{E}_k - \mathcal{E}_i) \frac{(p_k - p_i)}{2A_\Delta},
\]

where \( A_\Delta \) is the area of the triangle \( \Delta \) and \( \perp \) denotes a counterclockwise
rotation by 90° in the plane generated by the triangle, see [4]. The
gradient for every vertex is obtained by transforming and averaging the
gradients of the triangles. This yields the vertex gradient of the scalar
field \( \mathcal{E} \).

The vertex gradients of the scalar field can be used to calculate the
new normals of the carving structure. Given is a scalar field \( \mathcal{E} \) defined
on a plane by \( \mathcal{E} = f(x,y), x,y \in \mathbb{R} \). It generates a height surface by
setting \( z = f(x,y) = 0 \). Then, the unit normal vector \( n(x_0,y_0) \) at \((x_0,y_0)\)

\[
n(x_0,y_0) = \frac{1}{\sqrt{1 + (\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2}} \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{pmatrix}.
\]

This can be simplified by determining the gradient of \( f \): \( \nabla g = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}) \) then, the normal is

\[
n(x_0,y_0) = \frac{1}{\sqrt{1 + (\nabla g \cdot \nabla g)}} \begin{pmatrix} -\nabla g_x \\ -\nabla g_y \end{pmatrix}.
\]

As we considered the height field on the plane, the normal of this plane
is \( n_p = (0,0,1)^T \). Therefore, we can determine the angle \( \gamma \) between
the plane normal \( n_p \) and the normal determined by the height surface
\( n(x_0,y_0) \):

\[
\gamma = \arccos((n_p, n(x_0,y_0))) = \arccos\left(\frac{1}{\sqrt{1 + (\nabla g \cdot \nabla g)}}\right).
\]

Thus, this \( \gamma \) depends on the length of the gradient only. Hence, we
can calculate the normals of the generated carvings with the scalar
field. After we determined the vertex gradients, we used the smoothed normal to obtain the normals at the carvings. We apply Eq. 7, which yields the angle. This angle is used to determine how much we need to rotate the smoothed normal towards the vertex gradient in order to get the normal \( \mathbf{n}_D \) at the carving. Instead of using \( \mathbf{n}_D \), we mix it with the original normals. Otherwise, we would obtain unnatural looking carvings. We use a linear transformation of \( \mathbf{n}_D \) and \( \mathbf{n} \) to obtain the new normal \( \mathbf{n}' = \mathbf{C} \cdot \mathbf{n}_D + (1 - \mathbf{C}) \cdot \mathbf{n} \), where \( \mathbf{C} \) denotes the scalar field obtained by the drawing on the surface \( f(r) \). Thus, we obtain a more natural looking carvings, see Fig. 7.

6 Evaluation with Domain Experts

To assess the utility of the proposed methods, we conducted an informal evaluation with three domain experts. Two of them are monument conservators and one is an expert in the digitization of cultural heritage sites and artifacts. The evaluation was conducted in three steps. First, the participants answered general questions concerning their experience in this field, and provided some personal data. In the second step, we explained to the experts the possibilities that the proposed framework offers. Then, they could use the software for analyzing the data set, selecting and editing carvings, and generating new carvings. During this process, we encouraged them to think out loud and recorded their comments. After testing the tool, the participants filled out a questionnaire concerning their assessment of the usefulness of the tool for their work and their impression of the usability and handling of the software. In the last step, we asked them to mention positive as well as negative aspects about our tool.

General Aspects The general aspects comprise questions about age, gender, and experience. The participants (P1, P2, P3) are 65, 38, and 41 years old, respectively. P1 and P3 are males and P2 is female. (P1,P2) work as monument conservators and have experience for 46 and 20 years, respectively. (P3) is working for an institute that regularly acquires 3D data from cultural heritage objects. He has 20 years of experience. He is also responsible for preparing the data before the monument conservator can use it. Furthermore, we asked about the usefulness of having a tool that identifies the carvings. We also inquired about the participants’ background in computer-assisted analysis of carvings and about their experience in analyzing 3D digitizations of carvings. For most of the questions, we prepared 5-point Likert scale answers (+, +, 0, −, −−). The question about the importance of the analysis of the carvings using digital 3D models was answered with ‘++’ by all participants. The last question in this category was about the importance of the analysis of the carvings with our tool, which was rated with ‘++’ by P2 and P3 and with ‘+’ by P1. Framework Aspect After giving the participants the opportunity to use our tool, we asked several questions concerning the usefulness and handling of our tool. In particular, we asked about the possibilities for highlighting carvings and about the editing of the carvings. All participants stated that enhancing the visibility of the carvings is useful. P1 and P2 rated it with ‘++’ and P3 with ‘+’. Next, we inquired whether the functionality to delete parts of the detected carvings is helpful. All participants rated this with ‘++’. Another feature is that the illumination of the digital plaster can be modified by the positions and directions of the light sources to improve the perception of the carving structures. This feature was rated as useful and all participants judged it with ‘+’. All participants also confirmed the usefulness of the tool for creating new carvings. Here, P1 and P3 rated it with ‘++’ and P2 with ‘+'. The question whether the participants find the framework useful for analyzing the carvings was confirmed with ‘++’ by P1 and P2 and with ‘+’ by P3. Finally, we wanted to know if our method can reliably detect the carvings. P1 and P2 rated it with ‘+’, whereas P3 refrained from answering as he does not have enough knowledge about the datasets.

Comments In the last stage of the evaluation, we asked whether the experts would use our framework for analyzing the carvings in a professional setting. All participants replied with ‘yes’. The question whether they could use the tool for exploration of digitized plasters was also answered with ‘yes’ by all three participants. Finally, we asked the experts for additional feedback and for ideas and suggestions for additional features. They emphasized that some annotation tools would be helpful for sharing results with other domain experts and for working collectively on a dataset. Moreover, the experts would like to have a history listing who made which changes and the possibility to restrict the user’s access, such that an inexperienced user could only analyze but not edit the data.

7 Results

The plaster we consider is on a wall with width of 2.22 m and height of 0.98 m. After digitization of the plaster, the data comprises around 10 million vertices and around 21 million triangles. To handle the data set, we split the digitalized plaster in three parts and analyze them separately. For the sake of efficiency, we did not work on the fully resolved data set in most experiments. To create lower resolutions, we simplified the mesh using Edge Contraction with Quadric Error Metrics [11].

Resolution experiments The experiments were performed on a mid-class notebook with an Intel Core i5 with 2.6 GHz, 8 GB RAM, and an nVidia GeForce 870M with 4 GB. In Table 1, we show run times for different resolutions of the dataset. The preprocessing includes the curvature estimation, the computation of the vesselness measure and the determination of the ordered connected components of the carvings. As expected, the run times are linear to the number of vertices. On different datasets with approximately the same number of vertices, we obtained similar run times. Figure 8 shows a comparison of results obtained using different resolutions of the mesh.

Robustness experiments To test the robustness, we applied our method to a parametric surface: \( z = 2^{-k} \cos (2 \cdot \sqrt{x^2 + y^2}) \), with \( x,y \in [-10,10] \). Depending on \( k \), the height varies ranging from \( -1/2^k \) to \( 1/2^k \). For different \( k \), we tested if our algorithm can detect the carved structures, see Fig. 9 top row. The surface is depicted as well as the detected carvings top left. Thus, for different heights the carvings can be detected well. Afterwards, we added noise in normal direction, i.e.,

<table>
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<tr>
<th># Vertices</th>
<th>85k</th>
<th>40k</th>
<th>20k</th>
<th>10k</th>
<th>5k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>198</td>
<td>95</td>
<td>48</td>
<td>24</td>
<td>12</td>
</tr>
</tbody>
</table>
lead to misinterpretations. These paintings were used to show what the carvings on the wall looked like. Mostly, experts relied on these wrongly painted illustrations. Our domain experts confirmed that these illustrations give rise to false interpretations and that our approach is quite useful to detect the carvings well. They also stated that they want to use our results to publish new insights in the future.

**Different datasets.** For further evaluation of our approach, we applied our method to different datasets. We like to acknowledge the Laboratory of Computer Graphics & Multimedia at the Technion and to the Zinman Institute of Archaeology, University of Haifa for providing us the datasets. In Figure 11, we applied our method to different datasets. Thus, not only carvings can be detected and visualized, but also the letters and details on the ancient stone plates can be enhanced.

### 8 Comparison to Alternative Methods

We compared our approach with the exaggerated shading technique by Rusinkiewicz et al. [32] and with the D.o.R System [1], which is based on the work of Gilboa et al. [12] and Kolomenkin et al. [19]. The exaggerated shading emphasizes the features in the rendered surfaces well, and thereby highlights the carvings. However, since the technique highlights any surface feature it also emphasizes other parts in the digital plaster like noise or irregularities in the plaster. In contrast, our method separates the noise and natural structures in the plaster from the carvings and emphasizes only the carvings. Figure 12 shows a comparison of the plaster visualized with both approaches. Closest to our work is the method presented by Zatzarinni et al. [40]. Their technique works well for coarser carvings, but in case of strong noise and slightly bended surfaces their method cannot fully extract the carvings. For extracting the carvings, they proposed an automatic approach to set the threshold for distinguishing between foreground and background structures. Since this approach may be error-prone with noisy data, we determined the threshold manually to obtain more reliable results. Both our and Zatzarinni et al.’s approach are representing the surface containing the carvings using a height field over a base surface. To better explain why our approach gives the expected result, we visualize the height fields and the components, see Figure 13 and 14. Whereas our approach detects the carvings correctly, the approach by Zatzarinni et al. results in transition of the height from left to right due to the noisy data and the slightly bended surface. Even if we set different thresholds, not all carvings can be detected.
Fig. 12. Exaggerated shading [32] applied to the dataset (left) and the DoR System [1] (middle). Details can be depicted well, but noise is also enhanced. With our method (right) the carvings (shown in the inset on the figure) are detected and can be used to decently enhance the features.

Fig. 11. Various models with normal shading (left) and our method (right). The last two models show the identified carvings in different colors.

In addition to the comparison to alternative exaggerated shading techniques and the method by Zatzarinni et al., we tried feature line techniques to test whether these can be used for extracting and visualizing the carving structures. Our experiments indicate that such methods are too sensitive to noise to compete with the presented method for the extraction of carving structures. Also, enhancing the carvings with feature line methods did not yield results comparable to the proposed scheme. Examples of this kind are shown in Figure 15. The images illustrate that some features can be depicted well, but in all cases noise is also enhanced.

9 Discussion & Future Work

We present techniques to support monument restorers working with heritage-protected carving structures in plasters. Our methods enable the analysis, processing and visualization of carving structures in digital surfaces. We propose a method for extracting carving structures in surfaces based on the Frangi filter from image processing. The idea is to search in the surface for curvature configurations with a vanishing and a positive principal curvature. This concept is implemented by formulating a vesselness measure based on the principal curvatures. Furthermore, we propose techniques that support the user in finding appropriate parameters for the detection process. To robustly extract carvings, a connected component analysis is used in combination with a saliency measure for the components. The component analysis groups regions with the desired curvature pattern in connected carving structures. The saliency measure allows for ranking the groups. Setting a threshold on the saliency allows to robustly extract carvings. The component analysis ensures that coherent structures are obtained and noise is filtered out since it receives a low saliency value.

The carving structures are used as input for our visualization which highlights the carving structure and thereby increases their visibility. The resulting tool allows users to better perceive the carvings, in particular in cases where the structures have flattened due to aging and weathering. Moreover, the user can interact with the carving se-
Fig. 14. In (a) a dataset with Phong shading is shown. We manually enhanced the carvings to perceive more easily the features (b). In (b) and (e) the technique by Zatzarinni et al. is applied. Setting different thresholds will either detect the carvings left or right. In (c) and (f) our technique with a threshold is shown and it detects the carvings correctly.

Fig. 15. Different feature line methods are applied to the model. From left to right: shading, ridges and valleys, suggestive contours, apparent ridges, and photic extremum lines.

lection process by adjusting a simple and effective set of thresholds. Furthermore, users may wish to add new carvings. We propose an example-based approach for creating carvings in the “style” of the historic original carvings.

We confirmed the robustness of our approach by applying our techniques to artificially generated surfaces with different heights. With our method we were able to show what the carvings look like without risking misinterpretation from artists. As future work, the experts asked for several extensions. In particular, they asked for annotation tools. This would enable collaborative editing and exploration. Moreover, the experts would like to have a log listing who made which changes. The experts also suggested to extend our framework such that datasets with textures are supported. Then, the analysis could also take the texture into account, in addition to the current geometric analysis.

Altogether, based on the evaluation and our experiments, we consider our work a successful step in the development of visualization techniques and tools for supporting monument restorers.

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